Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns

A primary concern in calculating the critical axial buckling load P_c is the choice of the stiffness that reasonably approximates the variation in stiffness due to cracking, creep, and concrete nonlinearity. $(EI)_{eff}$ is used in the process of determining the moment magnification at column ends and along the column length in sway and nonsway frames.

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **ACI 318-14 (6.6.4.4.2)**

$$\delta = \frac{C_m}{1-\dfrac{P_u}{0.75P_c}} \geq 1.0 \text{ (For nonsway frames)}$$ **ACI 318-14 (6.6.4.5.2)**

$$\delta_s = \frac{1}{1-\dfrac{\Sigma P_u}{0.75\times\Sigma P_c}} \quad \text{(For sway frames)}$$ **ACI 318-14 (6.6.4.6.2(b))**

ACI 318-14 provides three options to calculate $(EI)_{eff}$ as follows:

$$(EI)_{eff} = \begin{Bmatrix} \text{(a)} & \dfrac{0.4E_cI_g}{1+\beta_{dns}} \\ \text{(b)} & \dfrac{0.2E_cI_g + E_sI_{se}}{1+\beta_{dns}} \\ \text{(c)} & \dfrac{E_cI}{1+\beta_{dns}} \end{Bmatrix}$$ **ACI 318-14 (6.6.4.4.4)**

AASHTO code provides two options to calculate EI for compression members in bridge structures as follows:

$$EI = \begin{Bmatrix} \text{(a)} & \dfrac{\dfrac{E_cI_g}{5}+E_sI_s}{1+\beta_d} \\ \text{(b)} & \dfrac{\dfrac{E_cI_g}{2.5}}{1+\beta_d} \end{Bmatrix}$$ **AASHTO 4th Edition (5.7.4.3)**

The moment of inertia of the section, I, in Eq. 6.6.4.4.4(c) is calculated per the formula in ACI 318, Table 6.6.3.1.1(b) for an individual column. It is incorrect to use I values from ACI 318, Table 6.6.3.1.1(a) in ACI 318, Eq. 6.6.4.4.4(c) as I values in ACI 318, Table 6.6.3.1.1(a) are intended to represent an overall average of moment of inertia values of EI for each member type which are used to compute frame deflections.

$$0.35I_g \leq I \leq 0.875I_g$$

Where $I = \left(0.80 + 25\frac{A_{st}}{A_g}\right) \times \left(1 - \frac{M_u}{P_u h} - 0.5\frac{P_u}{P_o}\right) \times I_g$ ***ACI 318-14 (Table 6.6.3.1.1(b))***

spColumn Program utilizes Eq. 6.6.4.4.4(b) for the calculation of the effective flexural stiffness, $(EI)_{eff}$, of column section. The other two equations, namely, Eq. 6.6.4.4.4(a), and Eq. 6.6.4.4.4(c) are also permitted by the ACI 318.

Comparison of $(EI)_{eff}$ values for Individual Columns

ACI 318 states that Eq. 6.6.4.4.4(a) is a simplified form of Eq. 6.6.4.4.4(b) and therefore, is less 'accurate'. If the reinforcing steel is not yet chosen, I_{se} cannot be computed and Eq. 6.6.4.4.4(a) is the only option to compute an initial value for $(EI)_{eff}$. On the other hand, ACI 318 states that Eq. 6.6.4.4.4(c) provides improved accuracy in $(EI)_{eff}$ calculation. However, a more complex formula of moment of inertia, I, is required. In that formula, P_u, and M_u values from each load combination must be considered. Alternatively, enveloped values of P_u, and M_u can be used conservatively to compute the lowest value of I.

A different value for the magnitude of the magnified moment is possible for each option of $(EI)_{eff}$. So which equation will lead to the optimum column design based on the ACI code provisions? To answer this question equation (a) is set equal to equation (b) as follows:

$$\frac{0.4E_c I_g}{1+\beta_{dns}} = \frac{0.2E_c I_g + E_s I_{se}}{1+\beta_{dns}}$$

This will lead to the relative stiffness non-dimensional factor α to be used to illustrate the comparison:

$$\alpha = \frac{E_s I_{se}}{E_c I_g} = 0.2$$

When α is greater than 0.2, $(EI)_{eff}$ obtained from equation (b) will be greater than $(EI)_{eff}$ obtained from equation (a) resulting in a lower value for the magnification factor and a lower magnified moment.

The Tables in the Appendix show the magnification factor (δ_s) for the design column in the "Sway Frame" example. It can be observed from the tables that when using 8-#11 bars, both equations yield the same $(EI)_{eff}$ and moment magnification factors. When smaller bars are used, equation (a) yields smaller values for the moment magnification factor. When larger bars are used, equation (b) yields smaller magnitude for the moment magnification factor.

Conclusions

The designer may utilize any of the options for *$(EI)_{eff}$* provided in ACI 318, section 6.6.4.4.4 given the available input data at a given design stage and the desired level of accuracy. spColumn Program utilizes Eq. 6.6.4.4.4(b) which is more accurate than Eq. 6.6.4.4.4(a) and less accurate (but less complex) than Eq. 6.6.4.4.4(c). The designer may choose between equations a and b to optimize the required moment magnification and finalize column size and reinforcement.

For bridge designer using the AASHTO the values of *EI* are given as the greater of:

$$EI = \begin{Bmatrix} \text{(a)} & \dfrac{\dfrac{E_c I_g}{5} + E_s I_s}{1+\beta_d} \\ \text{(b)} & \dfrac{\dfrac{E_c I_g}{2.5}}{1+\beta_d} \end{Bmatrix}$$

AASHTO 4th Edition (5.7.4.3)

References

[1] Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14), American Concrete Institute, 2014

[2] Wight J.K., Reinforced Concrete, Mechanics and Design, Seventh Edition, Pearson Education Inc., 2016

[3] Darwin D., Dolan C.W., Nilson A.H., Design of Concrete Structures, McGraw-Hill Education, 2016

Appendix

Comparison of Moment Magnification Factors Based 6.6.4.4.4 (a) and (b)

For an 18x18 column analyzed and designed for slenderness effects using spColumn (see slender column examples) a comparison for 8 bars of different sizes is briefly investigated to illustrate the impact of the equation choice on the magnitude of δ_s.

Table 1 – Moment Magnification Factors

bar size	δ_s - Using Equation 6.6.4.4.4(b)					
	δ_s - b (LC 4)	δ_s - b (LC 5)	δ_s - b (LC 6)	δ_s - b (LC 7)	δ_s - b (LC 8)	δ_s - b (LC 9)
#4	1.9	1.9	1.95	1.95	1.53	1.53
#5	1.8	1.8	1.85	1.85	1.49	1.49
#6	1.71	1.71	1.75	1.75	1.44	1.44
#7	1.63	1.63	1.66	1.66	1.4	1.4
#8	1.55	1.55	1.58	1.58	1.35	1.35
#9	1.49	1.49	1.51	1.51	1.32	1.32
#10	1.42	1.42	1.44	1.44	1.28	1.28
#11	1.37	1.37	1.39	1.39	1.25	1.25
E_cI_g, lb.in^2	δ_s - Using Equation 6.6.4.4.4(a)					
	δ_s - a (LC 4)	δ_s - a (LC 5)	δ_s - a (LC 6)	δ_s - a (LC 7)	δ_s - a (LC 8)	δ_s - a (LC 9)
8.62E+10	1.37	1.37	1.39	1.39	1.25	1.25